Concentration inequalities in the Fock space

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Given a certain notion of concentration on a function space, one may wonder under what conditions such concentration is optimal. This question immediately brings to mind the uncertainty principles considered in harmonic (Fourier) analysis, quantum mechanics, and signal processing.

If we consider the Fock space of entire functions that are square-integrable with respect to a Gaussian measure, and define the concentration of a function on a subset as the fraction of its norm concentrated in that subset, its optimizers were characterized by Nicola and Tilli [4]. Roughly speaking, among all subsets in the complex plane of a prescribed size and all functions in the Fock space, concentration is maximized when the subset is a disc and the function is a reproducing kernel.

The previous result can be reformulated in terms of the energy concentration of the short-time Fourier transform. Moreover, it is closely related to the study of the optimality of the Wehrl entropy, which began in the late 1970s. Lieb [2] proved that the Wehrl entropy of quantum Glauber states is minimized by coherent states. More recently, generalizations of Lieb's result—together with the characterization of the optimizers and the stability of the corresponding inequalities—have been successfully addressed.

In this workshop, we will become familiar with concentration inequalities in terms of both localization in a subset and (generalized) Wehrl entropy, and we will study some results related to the Fock space. Moreover, we will explore tools that have also been applied to analogous questions involving other spaces of holomorphic functions.

References

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[3] Kulikov, Nicola, Ortega-Cerdà, Paolo, *A monotonicity theorem for subharmonic functions on manifolds*, Adv. Math. 479, Part A, 18 p. (2025).

[4] Nicola, Tilli, *The Faber-Krahn inequality for the short-time Fourier transform*, Invent. Math. 230, No. 1, 1-30 (2022).